

1/5/2020 - The problems based when all the previous ~~methods~~ cases are failed. Then we solve the problem for P.I. to convert in the form $\frac{1}{f(D)} e^{ax} \cdot v$

Problem 11 solve $\frac{dy}{dx} - 4 \frac{dy}{dx} + 3y = 2e^{3x}$

Ans we write $(D^2 - 4D + 3)y = 2e^{3x}$

A.E. is $m^2 - 4m + 3 = 0$

$\Rightarrow m^2 - 3m - m + 3 = 0$

$\Rightarrow m(m-3) - 1(m-3) = 0$

$\Rightarrow (m-1)(m-3) = 0$

$\therefore m = \text{either } 1 \text{ or } 3$

C.F. $y = C_1 e^x + C_2 e^{3x}$

For P.I. $y = \frac{2e^{3x}}{D^2 - 4D + 3}$

when replacing D by 3 then $y = \frac{2}{0} e^{3x}$, case (1) is failed.

Now $y = \frac{2}{D^2 - 4D + 3} e^{3x} \cdot 1$ Here $v=1$

$y = e^{3x} \frac{2}{(D+3)^2 - 4(D+3) + 3} \quad (1)$

$= e^{3x} \frac{2}{D^2 + 6D + 9 - 4D - 12 + 3} \quad (1)$

$= e^{3x} \frac{2}{D^2 + 2D} \quad (1)$

$$\Rightarrow y = e^{3x} \cdot \frac{2}{D(D+2)} \quad (1)$$

$$= e^{3x} \cdot \left[\frac{1}{D} - \frac{1}{D+2} \right] \quad (1)$$

$$= e^{3x} \cdot \left[\frac{1}{D} (1) - \frac{1}{2(1+\frac{D}{2})} (1) \right]$$

$$= e^{3x} \cdot \left[x - \frac{1}{2} \left(1 + \frac{D}{2} \right)^{-1} (1) \right]$$

$$= e^{3x} \left[x - \frac{1}{2} \left(1 - \frac{D}{2} + \frac{D^2}{4} - \dots \right) (1) \right]$$

$$= e^{3x} \left[x - \frac{1}{2} - 0 \right] = e^{3x} \left(x - \frac{1}{2} \right)$$

\therefore The complete solution is

$$y = c_1 e^x + c_2 e^{3x} + e^{3x} \left(x - \frac{1}{2} \right) \text{ Ans}$$

Problem (12) Solve $\frac{d^2 y}{dx^2} + 4y = 4 \tan x$

As given equation is written as

$$(D^2 + 4)y = 4 \tan x$$

A.E. is $m^2 + 4 = 0$

$$m^2 = -4 = 4i^2 = (2i)^2$$

$$m = \pm 2i = 0 \pm 2i$$

C.F. is $y = e^{0x} (A \cos 2x + B \sin 2x)$

P.I. $y = \frac{4}{D^2 + 4} \tan x$

$$f = \frac{1}{D^2 - 4i^2} \tan 2x$$

$$= \frac{1}{(D+2i)(D-2i)} \tan 2x$$

$$= \frac{1}{2} \left[\frac{1}{D-2i} - \frac{1}{D+2i} \right] \tan 2x$$

$$= \frac{1}{2} \frac{1}{D-2i} \tan 2x - \frac{1}{2} \frac{1}{D+2i} \tan 2x$$

$$= \frac{1}{2} \frac{1}{D-2i} e^{i2x} \cdot e^{-i2x} \tan 2x - \frac{1}{2} \frac{1}{D+2i} e^{-i2x} \cdot e^{i2x} \tan 2x$$

$$= \frac{1}{2} e^{i2x} \frac{1}{D+2i-2i} e^{-i2x} \tan 2x - \frac{1}{2} e^{-i2x} \frac{1}{D-2i+2i} e^{i2x} \tan 2x$$

$$= \frac{1}{2} e^{i2x} \frac{1}{D} (\cos 2x - i \sin 2x) \tan 2x - \frac{1}{2} e^{-i2x} \frac{1}{D} (\cos 2x + i \sin 2x) \tan 2x$$

$$= \frac{1}{2} e^{i2x} \frac{1}{D} \left(\frac{\sin 2x - i \sin^2 2x}{\cos 2x} \right) - \frac{1}{2} e^{-i2x} \frac{1}{D} \left(\frac{\sin 2x + i \sin^2 2x}{\cos 2x} \right)$$

$$= \frac{1}{2} e^{i2x} \left[\frac{-\cos 2x}{2} - i \int \frac{1 - \cos 2x}{\cos 2x} dx \right] - \frac{1}{2} e^{-i2x} \left[\frac{\cos 2x}{2} + i \int \frac{1 - \cos 2x}{\cos 2x} dx \right]$$

$$= \frac{1}{2} e^{i2x} \left[\frac{\cos 2x}{2} - i \int [\sec 2x - \cos 2x] dx \right]$$

$$= \frac{1}{2} e^{-i2x} \left[-\frac{\cos 2x}{2} + i \int [\sec 2x - \cos 2x] dx \right]$$

$$= \frac{1}{2} \left[\frac{-\cos 2x}{2} - i \left\{ \log \left| \tan \left(\frac{x}{2} + \frac{2x}{2} \right) \right| - \frac{\sin 2x}{2} \right\} \right]$$

$$- \frac{1}{2} e^{-i2x} \left[-\frac{\cos 2x}{2} + i \left\{ \log \left| \tan \left(\frac{x}{2} + \frac{2x}{2} \right) \right| - \frac{\sin 2x}{2} \right\} \right]$$

$$\begin{aligned}
 y &= -\left(\frac{e^{iLx} - e^{-iLx}}{2i}\right) \cos Lx + \left(\frac{e^{iLx} + e^{-iLx}}{2}\right) \sin Lx \\
 &\quad - \left(\frac{e^{iLx} + e^{-iLx}}{2}\right) \log \tan\left(\frac{\pi}{4} + x\right) \\
 &= -\cancel{\sin Lx \cos Lx} + \cancel{\cos Lx \sin Lx} \\
 &\quad - \cos Lx \log \tan\left(\frac{\pi}{4} + x\right) \\
 &= -\log \tan\left(\frac{\pi}{4} + x\right) \cdot \cos Lx
 \end{aligned}$$

\therefore The Complete Solution is

$$y = C.F. + P.I$$

$$y = A \cos Lx + B \sin Lx - \log \tan\left(\frac{\pi}{4} + x\right) \cos Lx$$

Ans

Note $e^{iLx} = \cos Lx + i \sin Lx$

$e^{-iLx} = \cos Lx - i \sin Lx$

Then $\cos Lx = \frac{e^{iLx} + e^{-iLx}}{2}$

and $\sin Lx = \frac{e^{iLx} - e^{-iLx}}{2i}$

— x —

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1/5/2020