(ii) $ya=b$ has unique solⁿ in G.

Let e be the identity element of group 'G' and a^{-1}, be inverse of a.

$\therefore \quad aa^{-1} = a^{-1}a = e$ —— ①

$\because \quad ax = b$ —— ②

$\Rightarrow a^{-1}(ax) = a^{-1}b$

$\Rightarrow (a^{-1}a)x = a^{-1}b$ (by associative law)

$\Rightarrow ex = a^{-1}b$ {using (1)}

$\Rightarrow x = a^{-1}b$ —— ③

Uniqueness:- If possible Let $x=c$ be another solⁿ of ②

$\because x=c$ be a solⁿ of ②

$\therefore ac=b$

$\Rightarrow a^{-1}(ac) = a^{-1}b$

$\Rightarrow (a^{-1}a)c = a^{-1}b$ (by A.L)

$\Rightarrow ec = a^{-1}b$ {By ①}

$\Rightarrow c = a^{-1}b$.

$\therefore$ solⁿ ③ is unique.

Now, we shall show that this solⁿ belongs to G.

$\because a \in G \Rightarrow a^{-1} \in G$

$\therefore a^{-1} \in G,\ b \in G \Rightarrow a^{-1}b \in G$ (by closure property)

Hence the eqⁿ ② has unique solⁿ $a^{-1}b$ in G.

The given eqⁿ is

group is abelian.

or,

If G be a group and $a = a^{-1}$ $\forall a \in G$ then P.T. G is abelian

Ans Let a, b be two arbitrary elements of group G.

By question

$a = a^{-1}$, $b = b^{-1}$

$\because$ $a, b \in G \Rightarrow ab \in G$ [by closure property]

$\therefore$ $(ab)^{-1} = ab$

$\Rightarrow$ $b^{-1}a^{-1} = ab$

$\Rightarrow$ $ba = ab$

$\Rightarrow$ G is abelian

2. Prove that the set of all cube roots of unity form an abelian group under ordinary multiplication.

Ans Let $G = \{1, \omega, \omega^2\}$, where $1, \omega, \omega^2$ are three cube roots of unity.

To prove G is abelian group under multiplication

The multiplication table of G as under :—

$\times$	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	$\omega^3=1$
ω^2	ω^2	1	$\omega^4=\omega$

$\{\because \omega^3=1\}$

(i) from the table we see all elements of the table are members of G. Therefore closure property hold in G.

(ii) Since the elements of G are either real or complex, s they must obey associative law.

(iii) from table, we see 1 is the identity element of G.
(iv) from table we see the inverses of $1, \omega, \omega^2$ are $1, \omega^2, \omega$ & and G is ~~inverse table~~. invertiable.
from (i) and (iv) its follows that G is a Group unde multiplication.
Since the elements of G are either real or complex s must obey commotative law and hence 'G' is also ab group.

) Prove that the set of 4^{th} roots of unity form ~~one~~ an ~~abigio~~ abel group under multiplication.
Let $G=\{1,-1,i,-i\}$, where $1,-1,i,-i$ are 4^{th} roots of unity.
To prove G is abelian group under multiplication.
The multiplication table 'G' is under :-

*	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	-1	1
-i	-i	i	1	-1

{ $a * a^{-1} = e$
& here $e = 1$ }

(i) from this table ~~are~~ we see all elementary of the table are the member of G so G is ~~not~~ closed.

(ii) Since the elements of 'G' are either real or complex so they must obey a associative laws.

(iii) From table we see 1 is identity element of G
(iv) from table we see the inverse of 1, -1, i, -i are 1, -1, -i, i resp. and so G is ~~inverse table~~. invertiable.

from (i) and (iv) its follows that G is a group under multiplication.
Since the elements of G either real or complex so they must obey the commutative low and hence G is also abelian group under multiplication.

Note — n^{th} roots of unity

$$(1)^{1/n} = (\cos 2\pi r + i \sin 2\pi r)^{1/n}$$

$$= \cos \frac{2r\pi}{n} + i \sin \frac{2r\pi}{n}, \quad r = 0, 1, 2, \text{---}, n-1$$

$$= e^{i 2r \frac{\pi}{n}}$$

where $r = 0, 1, 2, - - -, (n-1)$

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