

Some more problems of Total diff. Equation

Problem (22) solve $(2x + y^2 + 2zx)dx + 2xydy + x^2dz = du$

Ans: Here $P = 2x + y^2 + 2zx$, $Q = 2xy$, $R = x^2$

C.O.G. $P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right)$

$$= (2x + y^2 + 2zx)(0 - 0) + 2xy(2x - 2x) + x^2(2y - 2y)$$

$$= 0, \text{ is satisfied.}$$

Hence given equation is integrable.

Now $(2x + y^2 + 2zx)dx + 2xydy + x^2dz = du$

$$\Rightarrow 2x dx + y^2 dx + 2zx dx + 2xy dy + x^2 dz = du$$

$$\Rightarrow 2x dx + x^2 dz + 2 \cdot 2x dx + y^2 dx + x \cdot 2y dy = du$$

$$\Rightarrow 2x dx + d(x^2z) + d(y^2x) = du$$

$$\Rightarrow 2 \int x dx + \int d(x^2z) + \int d(y^2x) = \int du$$

$$\Rightarrow x^2 + x^2z + y^2x = u + C \quad \underline{\text{Ans}}$$

Problem (23) solve: $2x dx + 2y dy + (x^2 + y^2 + z^2) dz = 0$

Ans: Here $P = 2x$, $Q = 2y$, $R = x^2 + y^2 + z^2$

C.O.G. $P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right)$

$$= 2x(0 - 2y) + 2y(2x - 0) + (x^2 + y^2 + z^2)(0 - 0)$$

$$= -4xy + 4xy + 0 = 0 \text{ is satisfied. Hence}$$

given equation is integrable. Now $2x dx + 2y dy + (x^2 + y^2) dz + z^2 dz = 0$

$$\Rightarrow d(x^2) + d(y^2) + (x^2 + y^2) dz = -z^2 dz$$

$$\Rightarrow e^z d(x^2 + y^2) + (x^2 + y^2) e^z dz = -dz$$

$$\Rightarrow d\{e^z(x^2 + y^2)\} = -dz, \text{ Integrating we have}$$

$$\Rightarrow e^z(x^2 + y^2) = -z + C \Rightarrow (x^2 + y^2)e^z + z = C \quad \underline{\text{Ans}}$$