SAttadm
K.C.C.
PG - II

D . G

Paper C-205
16.4.2020

Problem (1) Find the fundamental magnitudes of the surface of revolution.

Ans Let r be the current point. Then P.V. of a point of the surface of revolution is

$$r = [u\cos v,\ u\sin v,\ f(u)]$$

Then $r_1 = [\cos v,\ \sin v,\ f']$

$$r_2 = [-u\sin v,\ u\cos v,\ 0]$$

$$r_{11} = [0,\ 0,\ f''],\quad r_{21} = [-\sin v,\ \cos v,\ 0]$$

$$r_{22} = [-u\cos v,\ -u\sin v,\ 0]$$

$$E = r_1^2 = \cos^2 v + \sin^2 v + f'^2 = 1 + f'^2$$

$$F = r_1 \cdot r_2 = -u\sin v\cos v + u\cos v\sin v + 0 = 0$$

$$G = r_2^2 = u^2\sin^2 v + u^2\cos^2 v + 0 = u^2$$

$$r_1 \times r_2 = \begin{vmatrix} i & j & k \\ \cos v & \sin v & f' \\ -u\sin v & u\cos v & 0 \end{vmatrix} = [-uf'\cos v,\ -uf'\sin v,\ u]$$

$$= u[-f'\cos v,\ -f'\sin v,\ 1]$$

$$\therefore H^2 = EG - F^2 = u^2(1+f'^2) - 0 = u^2(1+f'^2)$$

$$H = u\sqrt{1+f'^2}$$

$$N = \frac{r_1 \times r_2}{H} = \frac{\cancel{u}[-f'\cos v,\ -f'\sin v,\ 1]}{\cancel{u}\sqrt{1+f'^2}}$$

$$= \frac{[-f'\cos v,\ -f'\sin v,\ 1]}{\sqrt{1+f'^2}}$$

$$L = N \cdot r_{11} = \frac{[-f'\cos v,\ -f'\sin v,\ 1] \cdot [0,\ 0,\ f'']}{\sqrt{1+f'^2}}$$

$$= \frac{0 + 0 + f''}{\sqrt{1+f'^2}} = \frac{f''}{\sqrt{1+f'^2}}$$

$$M = N \cdot r_{12} = \frac{[-f'\cos v,\ -f'\sin v,\ 1] \cdot [-\sin v,\ \cos v,\ 0]}{\sqrt{1+f'^2}}$$

$$= 0$$

$$N = N \cdot r_{22} = \frac{[-f'\cos v, -f'\sin v, 1]\cdot[-u\cos v, -u\sin v, 0]}{\sqrt{1+f'^2}}$$

$$= \frac{f'u\cos^2 v + f'u\sin^2 v + 0}{\sqrt{1+f'^2}} = \frac{uf'}{\sqrt{1+f'^2}}$$

Ans.

Problem (2) Find the fundamental magnitudes of Monge's form of the surface.

Ans Let equation of the surface in Monge's form is $z = f(x, y)$. Let r be the position vector of any point of the surface. Then $r = [x, y, f(x,y)]$. Taking x and y as parameters.

We write, $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$, $r = \frac{\partial^2 z}{\partial x^2}$,

$$s = \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}, \quad t = \frac{\partial^2 z}{\partial y^2}.$$

Now $r_1 = \left[1, 0, \frac{\partial z}{\partial x}\right] = [1, 0, p]$

$$r_2 = \left[0, 1, \frac{\partial z}{\partial y}\right] = [0, 1, q]$$

$$r_{11} = \left[0, 0, \frac{\partial^2 z}{\partial x^2}\right] = [0, 0, r]$$

$$r_{12} = r_{21} = \left[0, 0, \frac{\partial^2 z}{\partial x \partial y}\right] = [0, 0, s]$$

$$r_{22} = \left[0, 0, \frac{\partial^2 z}{\partial y^2}\right] = [0, 0, t]$$

Now, $E = r_1 \cdot r_1 = [1, 0, p]\cdot[1, 0, p] = 1+p^2$

$$F = r_1 \cdot r_2 = [1, 0, p]\cdot[0, 1, q] = pq$$

$$G = r_2 \cdot r_2 = [0, 1, q]\cdot[0, 1, q] = 1+q^2$$

and $H^2 = EG - F^2 = (1+p^2)(1+q^2) - p^2q^2$

$$= 1 + p^2 + q^2 + p^2q^2 - p^2q^2$$

$$= 1 + p^2 + q^2$$

$$\therefore H = \sqrt{1+p^2+q^2}$$

S. Srivastava
16/4/2020

Again, $N = \frac{r_1 \times r_2}{H} = \frac{[1, 0, p] \times [0, 1, q]}{\sqrt{1+p^2+q^2}}$

$= \frac{[0-p, -q-0, 1-0]}{\sqrt{1+p^2+q^2}} = \frac{[-p, -q, 1]}{\sqrt{1+p^2+q^2}}$

$L = N \cdot r_{11} = \frac{[-p, -q, 1] \cdot [0, 0, r]}{\sqrt{1+p^2+q^2}} = \frac{r}{\sqrt{1+p^2+q^2}}$

$M = N \cdot r_{12} = \frac{[-p, -q, 1] \cdot [0, 0, s]}{\sqrt{1+p^2+q^2}} = \frac{s}{\sqrt{1+p^2+q^2}}$

$N = N \cdot r_{22} = \frac{[-p, -q, 1] \cdot [0, 0, t]}{\sqrt{1+p^2+q^2}} = \frac{t}{\sqrt{1+p^2+q^2}}$

Problem (3) Calculate the fundamental magnitudes for the right helicoid given by
$x = u\cos\phi$, $y = u\sin\phi$, $z = c\phi$.

Ans Let r be the current point. Then

$r = [u\cos\phi, u\sin\phi, c\phi]$.

Let us write $r_1 = \frac{\partial r}{\partial u}$, $r_2 = \frac{\partial r}{\partial \phi}$ etc.

Now $r_1 = [\cos\phi, \sin\phi, 0]$

$r_2 = [-u\sin\phi, u\cos\phi, c]$

$r_{11} = [0, 0, 0]$, $r_{12} = [-\sin\phi, \cos\phi, 0]$

$r_{22} = [-u\cos\phi, -u\sin\phi, 0]$

$\therefore E = r_1 \cdot r_1 = \cos^2\phi + \sin^2\phi = 1$

$F = r_1 \cdot r_2 = -u\sin\phi\cos\phi + u\cos\phi\sin\phi + 0 = 0$

$G = r_2 \cdot r_2 = u^2\sin^2\phi + u^2\cos^2\phi + c^2 = u^2 + c^2$

$H^2 = EG - F^2 = u^2 + c^2 - 0 = u^2 + c^2$

$H = \sqrt{u^2 + c^2}$

Santashri
16/4/2020

$$N = \frac{r_1 \times r_2}{H} = \frac{-[-u\sin\phi, u\cos\phi, c] \times [\cos\phi, \sin\phi, 0]}{\sqrt{u^2+c^2}}$$

$$= \frac{[c\sin\phi, -c\cos\phi, u]}{\sqrt{u^2+c^2}}$$

$$L = N \cdot r_{11} = \frac{[c\sin\phi, -c\cos\phi, u] \cdot [0, 0, 0]}{\sqrt{u^2+c^2}} = 0$$

$$M = N \cdot r_{12} = \frac{[c\sin\phi, -c\cos\phi, u] \cdot [-\sin\phi, \cos\phi, 0]}{\sqrt{u^2+c^2}}$$

$$= \frac{-c\sin^2\phi - c\cos^2\phi + 0}{\sqrt{u^2+c^2}} = \frac{-c}{\sqrt{u^2+c^2}}$$

$$N = N \cdot r_{22} = \frac{[c\sin\phi, -c\cos\phi, u] \cdot [-u\cos\phi, -u\sin\phi, 0]}{\sqrt{u^2+c^2}}$$

$$= \frac{-uc\sin\phi\cos\phi + uc\sin\phi\cos\phi + 0}{\sqrt{u^2+c^2}} = 0$$

Problem (4) Calculate the fundamental magnitudes Ans and normal to the surface $2z = ax^2 + 2hxy + by^2$, taking x, y as parameters.

Ans Let r be the position vector of any point on the surface.

Then $r = (x, y, z) = \left(x, y, \frac{a}{2}x^2 + hxy + \frac{b}{2}y^2\right)$

$\therefore \quad r_1 = [1, 0, ax + hy]$

$r_2 = [0, 1, hx + by]$

$r_{11} = [0, 0, a]$, $r_{12} = [0, 0, h]$

$r_{22} = [0, 0, b]$

$$N = \frac{r_1 \times r_2}{H} = \frac{[1, 0, ax+hy] \times [0, 1, hx+by]}{H}$$

SSAHaque 16/4/20

$$N = \frac{[-(ax+hy), -(hx+by), 1]}{H}$$

Now, $E = r_1 \cdot r_1 = 1 + (ax+hy)^2$

$$F = r_1 \cdot r_2 = (ax+hy)(hx+by)$$

$$G = r_2 \cdot r_2 = 1 + (hx+by)^2$$

$$H^2 = EG - F^2 = [1+(ax+hy)^2][1+(hx+by)^2] - (ax+hy)^2(hx+by)^2$$

$$= 1 + (hx+by)^2 + (ax+hy)^2 + \cancel{(ax+hy)^2(hx+by)^2} - \cancel{(ax+hy)^2(hx+by)^2}$$

$$\therefore H = \sqrt{1+(hx+by)^2+(ax+hy)^2}$$

Also, $L = N \cdot r_{11}$

$$= \frac{[-(ax+by), -(hx+by), 1] \cdot [0, 0, a]}{H}$$

$$= \frac{0+0+a}{\sqrt{1+(hx+by)^2+(ax+hy)^2}}$$

$$= \frac{a}{\sqrt{1+(hx+by)^2+(ax+hy)^2}}$$

$$M = N \cdot r_{12} = \frac{[-(ax+by), -(hx+by), 1] \cdot [0, 0, h]}{H}$$

$$= \frac{h}{\sqrt{1+(hx+by)^2+(ax+hy)^2}}$$

$$N = N \cdot r_{22} = \frac{[-(ax+by), -(hx+by), 1] \cdot [0, 0, b]}{H}$$

$$= \frac{b}{\sqrt{1+(hx+by)^2+(ax+hy)^2}}$$

16/4/2020